ECE 344

Microwave Fundamentals

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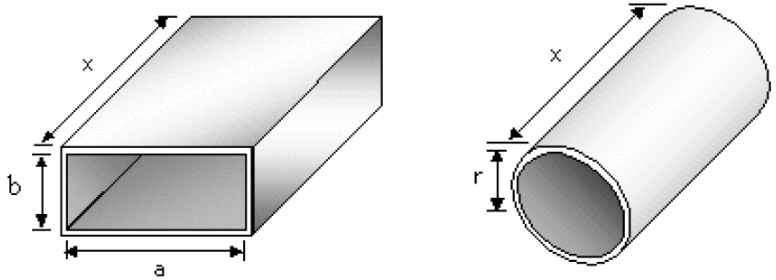
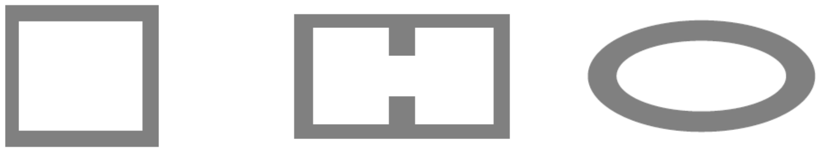
Waveguide Introduction

In general terms, a waveguide is a devise that confines electromagnetic energy and channels it from one point to another.

Examples



- Parallel plate waveguide
- Rectangular waveguide
- Circular waveguide
- Elliptical waveguide

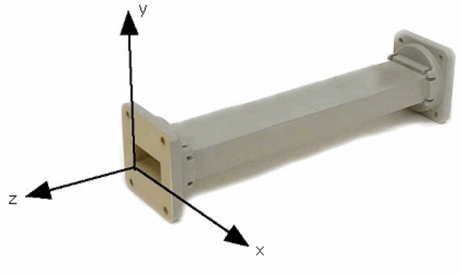


Square Waveguide

Double Ridge Rectangular Waveguide

Elliptical Waveguide

Note: In microwave engineering, the term "waveguide" is often used to mean rectangular or circular waveguide (i.e., a hollow pipe of metal).



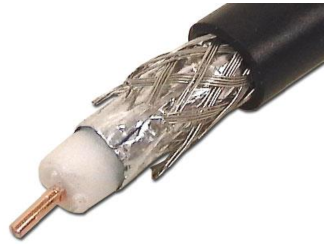
Comparison of Waveguide and Transmission Line Characteristics

T.L	W.G
Two or more conductors separated by some insulating medium (two-wire, coaxial, microstrip, etc.).	Metal waveguides are typically one enclosed conductor filled with an insulating medium (rectangular, circular) Dielectric waveguide consists of multiple dielectrics.
Normal operating mode is the TEM or quasi-TEM mode	Operating modes are TE or TM modes (cannot support a TEM mode).
No cutoff frequency for the TEM mode.	Must operate the waveguide at a frequency above the respective TE or TM mode cutoff frequency for that mode to propagate.
Significant signal attenuation at high frequencies due to conductor and dielectric losses.	Lower signal attenuation at high frequencies than transmission lines.
Small cross-section transmission lines (like coaxial cables) can only transmit low power levels	Metal waveguides can transmit high power levels.

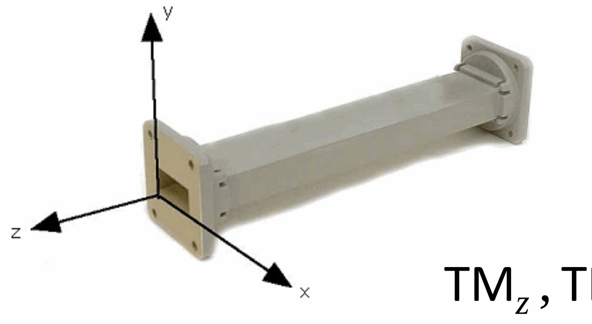
Field Representation (cont.)

Types of guided waves:

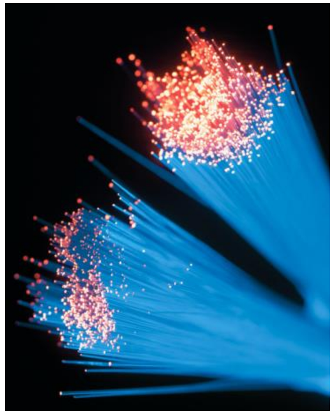
- TEM$_z$: $E_z = 0, \quad H_z = 0$
- TM$_z$: $E_z \neq 0, \quad H_z = 0$
- TE$_z$: $E_z = 0, \quad H_z \neq 0$
- Hybrid: $E_z \neq 0, \quad H_z \neq 0$



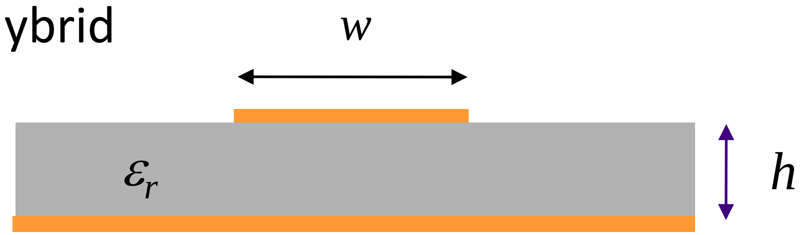
TEM$_z$



TM$_z$, TE$_z$



Hybrid



Microstrip

Modes in waveguides

Modes: certain field patterns that can propagate independently

- TEM mode: Transverse Electromagnetic mode

All the fields are in the cross section or there are no Ez and Hz components

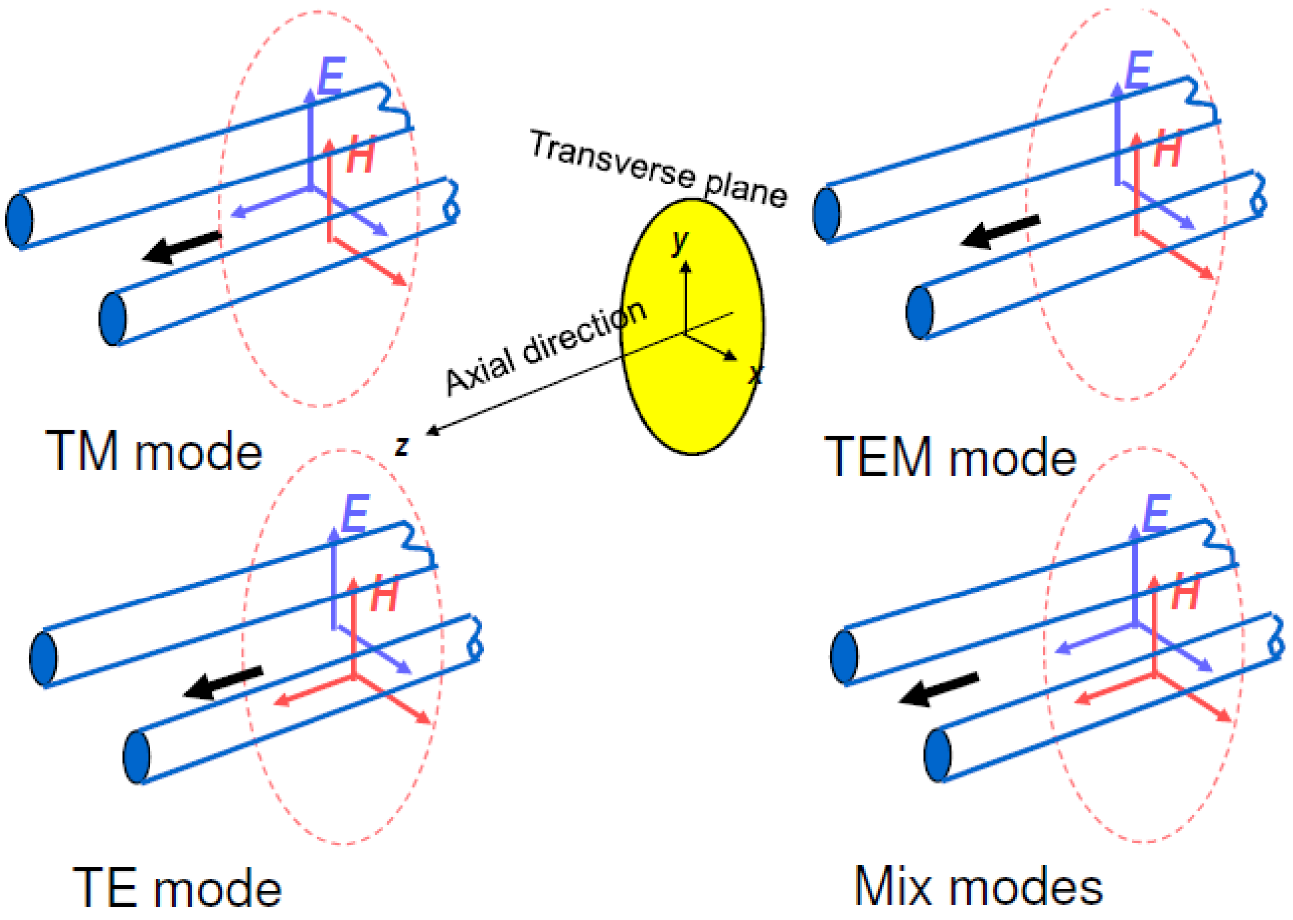
- TE modes: transverse electric modes

Electric fields are in the cross section (or no Ez component) Only Hz component in the longitudinal direction. Also called H modes

- TM modes: transverse magnetic modes

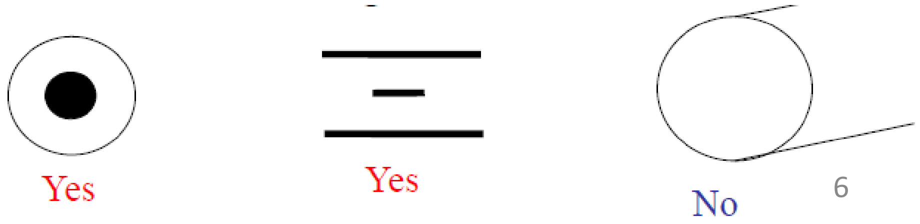
Magnetic fields are in the cross section (or no Hz component).Only Ez component in the longitudinal direction. Also called E modes

Propagation modes



Condition for the Existence of TEM Modes

- At least two perfect electric conductors



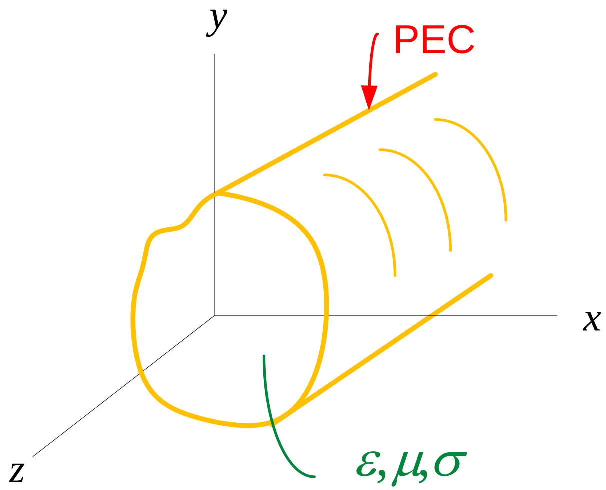
General Solutions for TEM, TE and TM Waves

Assume $e^{j\omega t}$ time dependence and homogeneous source-free materials.

Assume wave propagation in the $\pm z$ direction

$$\Rightarrow e^{\mp \gamma z} = e^{\mp jk_z z}$$

$$\gamma = jk_z, \quad k_z = \beta - j\alpha$$



$$\underline{J} = \sigma \underline{E}$$

$$\underline{E}(x,y,z) = \left[\underline{e}_t(x,y) + \hat{z}\, e_z(x,y)\right] e^{\mp jk_z z}$$

transverse components

$$\underline{H}(x,y,z) = \left[\underline{h}_t(x,y) + \hat{z}\, h_z(x,y)\right] e^{\mp jk_z z}$$

Field Representation: Proof

Assume a source-free region with a variation $e^{\mp jk_z z}$

$$\nabla \times \underline{E} = -j\omega\mu\underline{H} \qquad \nabla \times \underline{H} = j\omega\varepsilon_c\underline{E}$$

1) $\dfrac{\partial E_z}{\partial y} \pm jk_z E_y = -j\omega\mu H_x$

2) $\mp jk_z E_x - \dfrac{\partial E_z}{\partial x} = -j\omega\mu H_y$

3) $\dfrac{\partial E_y}{\partial x} - \dfrac{\partial E_x}{\partial y} = -j\omega\mu H_z$

4) $\dfrac{\partial H_z}{\partial y} \pm jk_z H_y = j\omega\varepsilon_c E_x$

5) $\mp jk_z H_x - \dfrac{\partial H_z}{\partial x} = j\omega\varepsilon_c E_y$

6) $\dfrac{\partial H_y}{\partial x} - \dfrac{\partial H_x}{\partial y} = j\omega\mu E_z$

Field Representation

Assume a guided wave with a field variation in the z direction of the form

$$e^{\mp jk_z z}$$

Then all six of the field components can be expressed in terms of these two fundamental ones:

$$\left(E_z, H_z\right)$$

Field Representation: Proof (cont.)

Combining 1) and 5)

1) $\dfrac{\partial E_z}{\partial y} \pm jk_z E_y = -j\omega\mu H_x$

5) $\mp jk_z H_x - \dfrac{\partial H_z}{\partial x} = j\omega\varepsilon_c E_y$

$$\Rightarrow \frac{\partial E_z}{\partial y} \pm jk_z \left(\frac{-1}{j\omega\varepsilon_c}\right)\left(\pm jk_z H_x + \frac{\partial H_z}{\partial x}\right) = -j\omega\mu H_x$$

$$\frac{\partial E_z}{\partial y} \mp \frac{k_z}{\omega\varepsilon_c}\frac{\partial H_z}{\partial x} = \left(-j\omega\mu - \frac{k_z^2}{j\omega\varepsilon_c}\right) H_x$$

$$\Rightarrow j\omega\varepsilon_c \frac{\partial E_z}{\partial y} \mp jk_z \frac{\partial H_z}{\partial x} = \underbrace{(k^2 - k_z^2)}_{k_c^2} H_x$$

$$\Rightarrow H_x = \frac{j}{k_c^2}\left(\omega\varepsilon_c \frac{\partial E_z}{\partial y} \mp k_z \frac{\partial H_z}{\partial x}\right)$$

$$k_c = \left(k^2 - k_z^2\right)^{1/2}$$

Cutoff wave number

A similar derivation holds for the other three transverse field components.

Field Representation (cont.)

$$H_x = \frac{j}{k_c^2}\left(\omega\varepsilon_c \frac{\partial E_z}{\partial y} \mp k_z \frac{\partial H_z}{\partial x}\right)$$

$$H_y = \frac{-j}{k_c^2}\left(\omega\varepsilon_c \frac{\partial E_z}{\partial x} \pm k_z \frac{\partial H_z}{\partial y}\right)$$

$$E_x = \frac{-j}{k_c^2}\left(\pm k_z \frac{\partial E_z}{\partial x} + \omega\mu \frac{\partial H_z}{\partial y}\right)$$

$$E_y = \frac{j}{k_c^2}\left(\mp k_z \frac{\partial E_z}{\partial y} + \omega\mu \frac{\partial H_z}{\partial x}\right)$$

These equations give the transverse field components in terms of *longitudinal* components, E_z and H_z.

$$k^2 = \omega^2 \mu\varepsilon_c$$

$$k_c = \left(k^2 - k_z^2\right)^{1/2}$$

Field Representation, Helmholtz Equation

Therefore, we only need to solve the Helmholtz equations for the longitudinal field components (E_z and H_z).

$$\nabla^2 E_z + k^2 E_z = 0$$

$$\nabla^2 H_z + k^2 H_z = 0$$

Transverse Electric (TE_z) Waves

$$\Rightarrow E_z = 0$$

The electric field is "transverse" (perpendicular) to z.

In general, $E_x, E_y, H_x, H_y, H_z \neq 0$

To find the TE_z field solutions (away from any sources), solve

$$(\nabla^2 + k^2) H_z = 0$$

$$(\nabla^2 + k^2) H_z = 0 \Rightarrow \left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2} + k^2 \right) H_z = 0$$

Transverse Electric (TE_z) Waves (cont.)

$$\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}+\frac{\partial^2}{\partial z^2}+k^2\right)H_z=0$$

Recall that the field solutions we seek are assumed to vary as

$$e^{\mp jk_z z}$$

$$\Rightarrow H_z(x,y,z)=h_z(x,y)\,e^{\mp jk_z z}$$

$$\Rightarrow\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}\underbrace{-k_z^2+k^2}_{k_c^2}\right)h_z(x,y)=0$$

$$k_c^2=k^2-k_z^2$$

$$\Rightarrow\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}+k_c^2\right)h_z(x,y)=0$$

← Solve subject to the appropriate boundary conditions.

Transverse Electric (TE_z) Waves (cont.)

Once the solution for H_z is obtained,

$$\Rightarrow H_x = \mp \frac{jk_z}{k_c^2}\frac{\partial H_z}{\partial x} \qquad E_x = \frac{-j\omega\mu}{k_c^2}\frac{\partial H_z}{\partial y}$$

$$H_y = \mp \frac{jk_z}{k_c^2}\frac{\partial H_z}{\partial y} \qquad E_y = \frac{j\omega\mu}{k_c^2}\frac{\partial H_z}{\partial x}$$

For a wave propagating in the positive z direction (top sign):

$$\frac{E_x}{H_y} = -\frac{E_y}{H_x} = \frac{\omega\mu}{k_z}$$

For a wave propagating in the negative z direction (bottom sign):

$$-\frac{E_x}{H_y} = \frac{E_y}{H_x} = \frac{\omega\mu}{k_z}$$

TE wave impedance

$$Z_{TE} = \frac{\omega\mu}{k_z}$$

Transverse Magnetic (TM_z) Waves

$$\Rightarrow H_z = 0$$

In general, $E_x, E_y, E_z, H_x, H_y \neq 0$

To find the TE_z field solutions (away from any sources), solve

$$(\nabla^2 + k^2) E_z = 0$$

$$(\nabla^2 + k^2) E_z = 0 \Rightarrow \left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2} + k^2 \right) E_z = 0$$

Transverse Magnetic (TM_z) Waves (cont.)

$$\Rightarrow \left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} \underbrace{- k_z^2 + k^2}_{k_c^2} \right) e_z(x, y) = 0$$

$$\Rightarrow \left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + k_c^2 \right) e_z(x, y) = 0$$

$$k_c^2 = k^2 - k_z^2$$

solve subject to the appropriate boundary conditions

Transverse Magnetic (TM_z) Waves (cont.)

Once the solution for E_z is obtained,

$$\Rightarrow H_x = \frac{j\omega\varepsilon_c}{k_c^2}\frac{\partial E_z}{\partial y} \qquad E_x = \mp\frac{jk_z}{k_c^2}\frac{\partial E_z}{\partial x}$$

$$H_y = -\frac{j\omega\varepsilon_c}{k_c^2}\frac{\partial E_z}{\partial x} \qquad E_y = \mp\frac{jk_z}{k_c^2}\frac{\partial E_z}{\partial y}$$

For a wave propagating in the positive z direction (top sign):

$$\frac{E_x}{H_y} = -\frac{E_y}{H_x} = \frac{k_z}{\omega\varepsilon_c}$$

For a wave propagating in the negative z direction (bottom sign):

$$-\frac{E_x}{H_y} = \frac{E_y}{H_x} = \frac{k_z}{\omega\varepsilon_c}$$

TM wave impedance

$$Z_{TM} = \frac{k_z}{\omega\varepsilon_c}$$

Transverse ElectroMagnetic (TEM) Waves

$$\Rightarrow E_z = 0,\ H_z = 0$$

In general, $E_x, E_y, H_x, H_y \neq 0$

From the previous equations for the transverse field components, all of them are equal to zero if E_z and H_z are both zero.

<u>Unless</u> $k_c^2 = 0$

$\Rightarrow$ For TEM waves $k_c^2 = k^2 - k_z^2 = 0$

Hence, we have

$$k_z = k = \omega\sqrt{\mu\varepsilon_c}$$

Rectangular waveguide

Need to find the fields components of the *em* wave inside the waveguide

$$E_z \; H_z \; E_x \; H_x \; E_y \; H_y$$

Equations3.5a to 3.5d express transverse components E_x H_x E_y H_y in term of longitudinal fields components E_z H_z

assuming waveguide filled with lossless dielectric material and walls of perfect conductor, the wave inside should obey wave equation:

$$\nabla^2 E + k^2 E = 0$$

$$\nabla^2 H + k^2 H = 0$$

$$\text{where} \quad k^2 = \omega^2 \mu \varepsilon$$

Applying to Z component in TE :

$$\nabla^2 H_z + k^2 H_z = 0 \longrightarrow$$

$$\frac{\partial^2 H_z}{\partial x^2} + \frac{\partial^2 H_z}{\partial y^2} + \frac{\partial^2 H_z}{\partial z^2} + k^2 H_z = 0 \rightarrow \frac{\partial^2 H_z}{\partial x^2} + \frac{\partial^2 H_z}{\partial y^2} + k_c^{\,2} H_z$$

$$where \frac{\partial^2}{\partial z^2} = (-j\beta)^2 \,, k_c^{\,2} = k^2 - \beta^2$$

Rectangular Waveguide

Method of solution:

1. Solve Helmholtz equation for either Hz (TE) or Ez (TM).

2. Solve for the constants from the boundary conditions. In metal boundaries which is tangential E = 0 on the boundary. Now you have Ez or Hz.

3. Use 3.19(for TE) or 3.23(for TM) to find transverse components from Ez or Hz.

TE Solution

1. Solve Helmholtz wave equation:

$$\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}+k_c^2\right)h_z(x,y)=0$$

a) Use Method of Separation of Variables:
$h_z(x,y) = X(x)\,Y(y)$

b) Substitute into wave equation (Helmholtz equation):

$$Y\frac{d^2X}{dx^2}+X\frac{d^2Y}{dy^2}+k_c^2XY=0$$

where $k_c^2 = k_x^2 + k_y^2$

c) Separate the Variables:

$$Y\left[\frac{d^2X}{dx^2}+k_x^2X\right]=0$$

$$X\left[\frac{d^2Y}{dy^2}+k_y^2Y\right]=0$$

d) "Guess" the form of the solution:

$h_z(x,y) = (A \cos k_x x + B \sin k_x x)\,(C \cos k_y y + D \sin k_y y)$

The unknown constants are ABCD. Also k_x and k_y.

2. Solve for the unknown constants from boundary conditions.

a) Define the boundary conditions
Tangential E fields =0 on the metal surfaces (walls of the waveguide)
$e_x = 0$ at $y=0,b$
$e_y = 0$ at $x=0,a$

b) Obtain appropriate expressions for the boundary condition fields
From equations 3.19:
$E_x = (-j\omega\mu / k_c^2)\, \partial H_z / \partial y$
$E_y = (j\omega\mu / k_c^2)\, \partial H_z / \partial x$

So:
$e_x(x,y) = (-j\omega\mu / k_c^2)\, k_y\ (A \cos k_x x + B \sin k_x x)\,(-C \sin k_y y + D \cos k_y y)$
$e_y(x,y) = (j\omega\mu / k_c^2)\, k_x\, (-A \sin k_x x + B \cos k_x x)\,(C \cos k_y y + D \sin k_y y)$

c) Use boundary conditions to solve for ABCD:

c) *Use boundary conditions to solve for A,B,C, and D:*

Substitute $e_x = 0$ *at* $y=0,b$ *into equations above.*

$e_x(x,0) = (-j\omega\mu / k_c^2)\, k_y\, (A \cos k_x x + B \sin k_x x)\, (- C \sin k_y 0 + D \cos k_y 0) = 0$ So, $\mathbf{D = 0}$

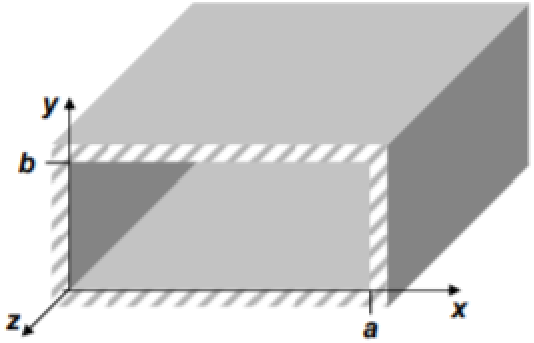
$e_x(x,b) = (-j\omega\mu / k_c^2)\, k_y\, (A \cos k_x x + B \sin k_x x)\, (- C \sin k_y b + 0 \cos k_y b) = 0$ $k_y = n\pi/b$

Substitute $e_y = 0$ *at* $x=0,a$ *into equations above:*

- $e_y(0,y) = (j\omega\mu / k_c^2)\, k_x\, (-A \sin k_x 0 + B \cos k_x 0)\, (C \cos k_y y + D \sin k_y y) = 0$ So, $\mathbf{B=0}$
- $e_y(x,a) = (j\omega\mu / k_c^2)\, k_x\, (-A \sin k_x a + B \cos k_x a)\, (C \cos k_y y + D \sin k_y y) = 0$ So, $k_x = m\pi/a$

Now we can simplify the form:

- $e_x(x,b) = (-j\omega\mu / k_c^2)\, (n\pi/b)\, (-AC) \cos(m\pi x/a) \sin (n\pi y/b)$
- $E_x = (j\omega\mu n\pi / b k_c^2)\, A_{mn} \cos(m\pi x/a) \sin (n\pi y/b)\, e^{-j\beta Z}$
- $E_y = (-j\omega\mu m\pi/a k_c^2)\, A_{mn} \sin (m\pi x/a) \cos (n\pi y/b)\, e^{-j\beta Z}$



$e_x(x,y) = (-j\omega\mu / k_c^2)\, k_y\, (A \cos k_x x +$ ~~$B \sin k_x x$~~$)(- C \sin k_y y +$ ~~$D \cos k_y y$~~$)$

$e_y(x,y) = (j\omega\mu / k_c^2)\, k_x\, (-A \sin k_x x +$ ~~$B \cos k_x x$~~$)(C \cos k_y y +$ ~~$D \sin k_y y$~~$)$

TE mode

$$H_z = A_{mn} \cos\frac{m\pi}{a} x \cos\frac{n\pi}{b} y \, .e^{-j\beta z}$$

$$H_x = \frac{-j\beta}{K_c^2}\frac{\partial Hz}{\partial x} = \frac{j\beta}{K_c^2} A_{mn} \frac{m\pi}{a} \sin\frac{m\pi}{a} x \cos\frac{n\pi}{b} y \, .e^{-j\beta z}$$

$$H_y = \frac{-j\beta}{K_c^2}\frac{\partial Hz}{\partial y} = \frac{j\beta}{K_c^2} A_{mn} \frac{n\pi}{b} \cos\frac{m\pi}{a} x \sin\frac{n\pi}{b} y \, .e^{-j\beta z}$$

we get at previous slide that

$$E_x = \frac{j\omega\mu n\pi}{K_c^2 b} A_{mn} \cos\frac{m\pi}{a} x \sin\frac{n\pi}{b} y \, .e^{-j\beta z}$$

$$E_y = \frac{-j\omega\mu m\pi}{K_c^2 a} A_{mn} \sin\frac{m\pi}{a} x \cos\frac{n\pi}{b} y \, .e^{-j\beta z}$$

$$E_z = 0$$

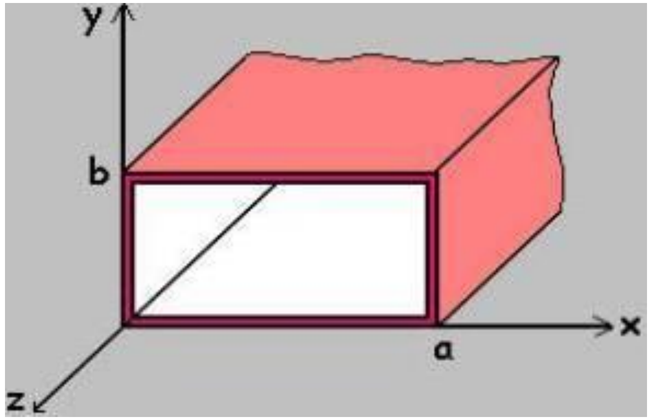
From these equations we found that:
TE_{00} does not exist as all E components=0 there
So either m or n allowed to be equal to 0.

TM modes has components fields m,n start from 1
i.E TM_{00} TM_{01} TM_{10} does not exist(page111)and lowest order TM mode to propagate is TM_{11}

$k_c{}^2 = k^2 - \beta^2$

$\beta = \sqrt{k^2 - k_c{}^2} \rightarrow$ *for* propagation β must be real or $k^2 > k_c{}^2$

or $f > f_c$ i.e operating frequency > cut off frequency



<u>Cut off frequency f_c :</u>

$$k_c{}^2 = \omega_c \sqrt{\mu\varepsilon} = k_x{}^2 + k_y{}^2 = \left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2$$

$$\rightarrow f_c = \frac{1}{2\pi\sqrt{\mu\varepsilon}} \sqrt{\left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2}$$

<u>phase velocity:</u>

$v_p = \frac{\omega}{\beta} > \frac{\omega}{\omega\sqrt{\mu\varepsilon}}$ phase velocity is greater than speed of light in filling material

wave impedance $Z_{TE} = \frac{E_x}{H_y} = \frac{k\eta}{\beta} = \eta_{TE}$

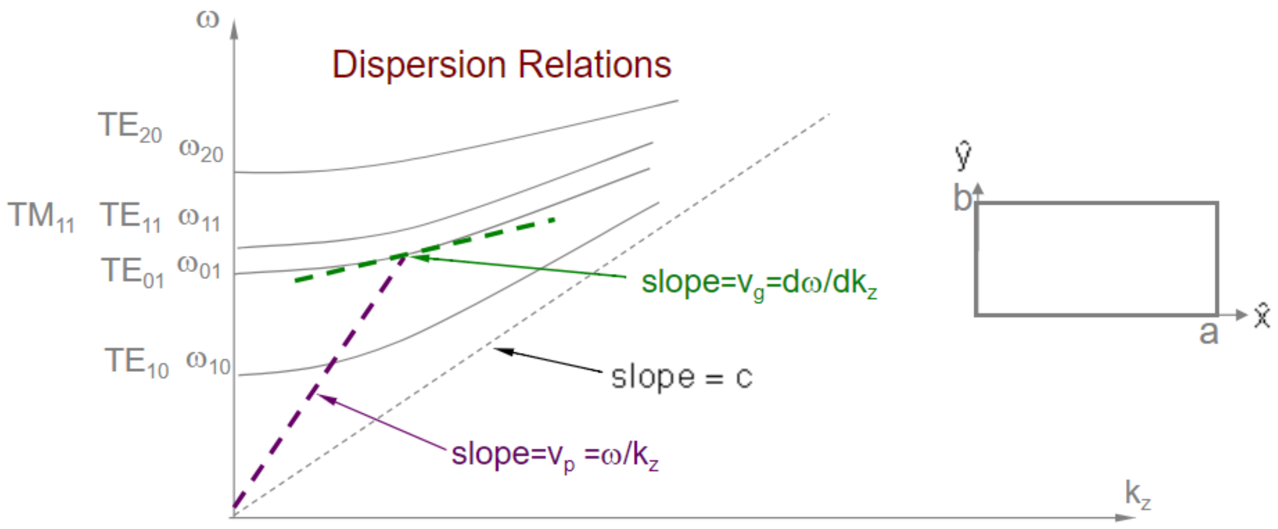
where η_{TE} is intrinsic impedance of the mode

η is intrinsic impedance of material filling waveguide $= \sqrt{\frac{\mu}{\varepsilon}}$

- phase velocity is **frequency dependent** so waveguides are dispersive.

Group velocity : Is the velocity at which the energy travels.

$$v_g = \frac{1}{\partial \beta / \partial \omega}$$



The guide wave length is defined by the distance between two equal phase planes along wave guide

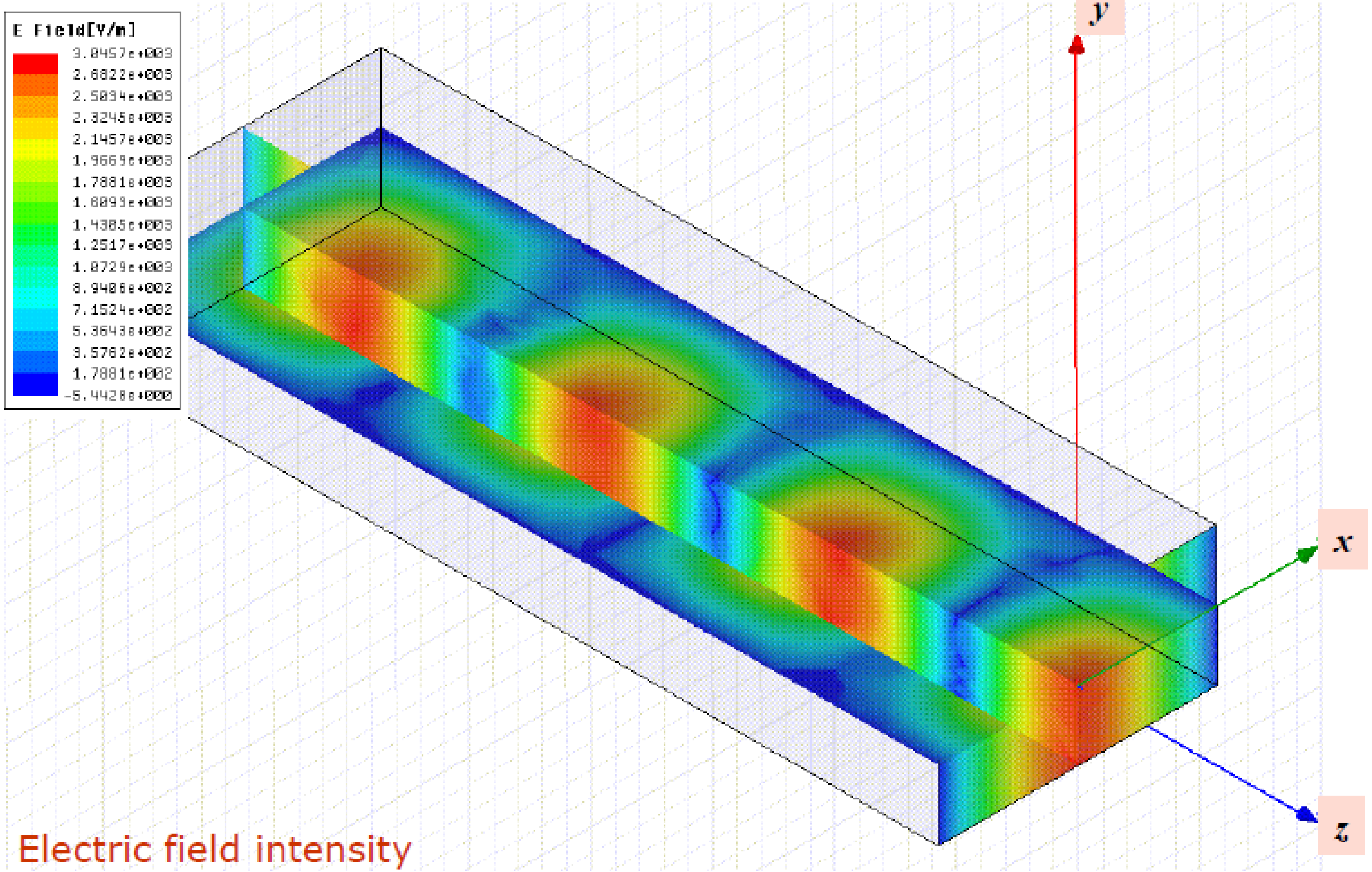
$$\lambda_g = \frac{2\pi}{\beta} = \frac{\lambda}{\sqrt{1 - \frac{f_c^2}{f^2}}} > \lambda$$

g for guide

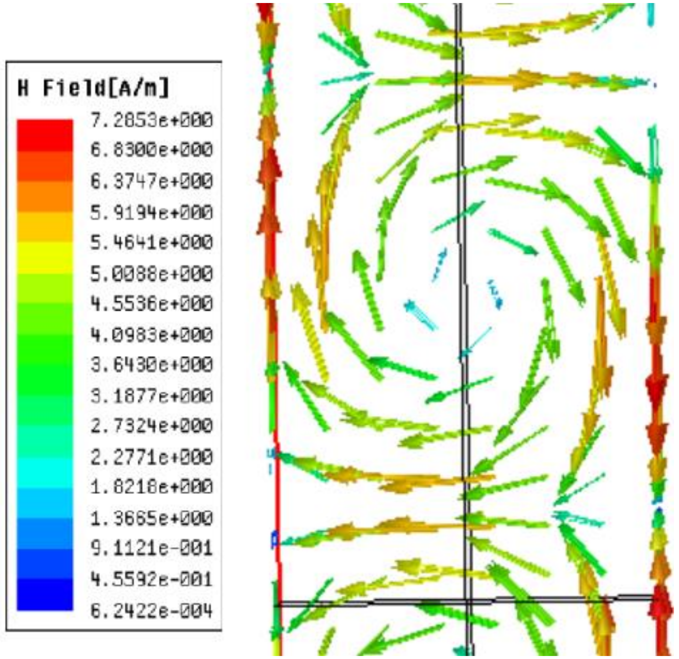
This is the "free space" wavelength

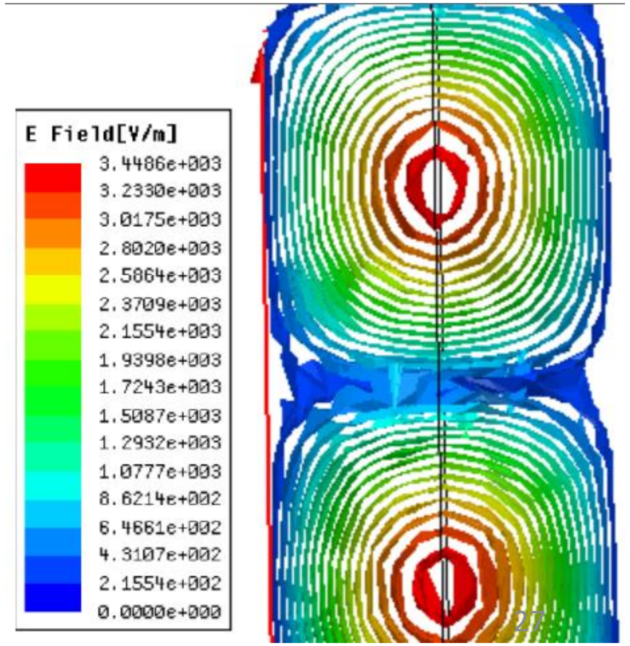
- The lowest cut off frequency called the dominant mode. Assume a>b dominant mode is TE_{10}
- Wave guide is said to be overmoded if more than one mode propagate

Rectangular waveguide dominant mode (TE_{10})



Electric field intensity





Waveguides

Here are some standard air-filled rectangular waveguides with their naming designations, inner side dimensions a, b in inches, cutoff frequencies in GHz, minimum and maximum recommended operating frequencies in GHz, power ratings, and attenuations in dB/m (the power ratings and attenuations are representative over each operating band.) We have chosen one example from each microwave band.

name	a	b	f_c	f_{min}	f_{max}	band	P	α
WR-510	5.10	2.55	1.16	1.45	2.20	L	9 MW	0.007
WR-284	2.84	1.34	2.08	2.60	3.95	S	2.7 MW	0.019
WR-159	1.59	0.795	3.71	4.64	7.05	C	0.9 MW	0.043
WR-90	0.90	0.40	6.56	8.20	12.50	X	250 kW	0.110
WR-62	0.622	0.311	9.49	11.90	18.00	Ku	140 kW	0.176
WR-42	0.42	0.17	14.05	17.60	26.70	K	50 kW	0.370
WR-28	0.28	0.14	21.08	26.40	40.00	Ka	27 kW	0.583
WR-15	0.148	0.074	39.87	49.80	75.80	V	7.5 kW	1.52
WR-10	0.10	0.05	59.01	73.80	112.00	W	3.5 kW	2.74

Characteristics of some standard air-filled rectangular waveguides.

Example 1 :

X-band rectangular waveguide X-band≈8-12GHz Air filled i.e. $\varepsilon r=1$

For a=2.29 cm , b=1.02cm
Find (a) first 8 mode of propagation (b) β and λg at 10GHz (c) β and λg at 6GHz

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(a)

mode	Fc [GHz]
TE10	6.55
TE20	13.1
TE01	14.71
TE11	16.1
TM11	16.1
TE30	19.65
TE21	19.69
TM21	19.69

(b) at f =10 GHz It propagate at TE10 mode

$$\beta = \sqrt{k^2 - k_c^2} = \sqrt{\frac{(2\pi*10^{10})^2}{(3*10^8)^2} - \left(\frac{\pi}{0.0229}\right)^2}\text{=158.25 rad/m}$$

$$\lambda_g = \frac{2\pi}{\beta}\text{=3.97 cm}$$

(c) at f=6 GHz

$$\beta = \sqrt{k^2 - k_c^2} = \sqrt{\frac{(2\pi*6*10^9)^2}{(3*10^8)^2} - \left(\frac{\pi}{0.0229}\right)^2}\text{=-j55.04}$$

it is converted to attenuation with α=55.04 Np/m

So no propagation and λ_g not defined

Example 2 : ***For fundamental mode***

A rectangular waveguide measures 3×4.5cm internally and has a 10 GHz signal propagated in it. Calculate the cut off frequency (λ_c) and the guide wavelength (λ_g).

Ans: $$\lambda_c = \frac{2}{\sqrt{\left(\frac{m}{a}\right)^2 + \left(\frac{n}{b}\right)^2}}$$

a=4.5cm

TE_{10} mode m = 1, n = 0

$$\lambda_c = \frac{2a}{m} = 2 \times 0.045 = 0.090m$$

$$\lambda_g = \frac{\lambda}{\sqrt{1 - \left(\frac{\lambda}{\lambda_c}\right)^2}} \quad \text{where} \quad \lambda = \frac{3 \times 10^8}{10 \times 10^9} = 0.03m$$

$$\lambda_g = 0.0318m.$$

$$\lambda_g = \frac{2\pi}{\beta} = \frac{2\pi}{\sqrt{k^2 - k_c^2}} = \frac{2\pi}{2\pi f \sqrt{\mu\varepsilon} \sqrt{1 - \left(\frac{f_c}{f}\right)^2}} = \frac{\lambda}{\sqrt{1 - \left(\frac{\lambda}{\lambda_c}\right)^2}}$$

Example 3 : In a rectangular waveguide for which a = 1.5 cm, b = 0.8 cm, $\sigma = 0$, $\mu = \mu_o$ and $\epsilon = 4\epsilon_o$ and

$$H_x = 2\sin\left(\frac{\pi x}{a}\right)\cos\left(\frac{3\pi y}{b}\right)\text{Sin}\left(\pi \times 10^{11} t - \beta z\right) A/m$$

Find

(i) the mode of operation
(ii) the cut off frequency
(iii) phase constant

Ans:

(i) TM_{13} or TE_{13}

(ii) $$f_{c_{mn}} = \frac{1}{2\sqrt{\mu\varepsilon}}\sqrt{\frac{m^2}{a^2}+\frac{n^2}{b^2}} = \frac{c}{4}\sqrt{\frac{m^2}{a^2}+\frac{n^2}{b^2}}$$

Or $f_{c_{13}} = 28.57 GH_z$

(iii) $$\beta = \omega\sqrt{\mu\epsilon}\sqrt{1-\left(\frac{f_c}{f}\right)^2}$$

$$= \frac{\omega\sqrt{\varepsilon r}}{C}\sqrt{1-\left(\frac{f_c}{f}\right)^2}$$

$\beta = 1718.81$ rad/m

Where $\omega = 2\pi f = \pi \times 10^{11}$ or $f = \frac{100}{2} = 50 GH_z$

Example 4 :

Find the following:

(i) the possible transmission modes in a hollow rectangular waveguide of inner dimension 3.44 × 7.22 cm at an operating frequency of 3000 MHz.

(ii) the corresponding values of phase velocity, group velocity and phase constant. (7)

Ans: Free space wave length, $\lambda_0 = \frac{c}{f} = \frac{3\times10^8}{3000\times10^6}$ metre

$= 10$ cm

Also, we know that the cut off wave length

$$\lambda_c = \frac{2}{\sqrt{\left(\frac{m}{a}\right)^2 + \left(\frac{n}{b}\right)^2}}$$

Possible modes:

(i) For TE_{00} mode i.e. m = 0, n = 0, $\lambda_c = \infty$, (i.e. $\lambda_c > \lambda_0$) and hence there will be no propagation.

(ii) For TE_{10} mode i.e. m = 1, n = 0 and $\lambda_c = 2a = 2\times 7.22 = 14.44$ cms. Hence this mode will propagate because $\lambda_c > \lambda_0$.

(iii) For TE_{01} mode i.e. m =0, n = 1, and $\lambda_c = 2b = 2\times 3.44 = 6.88$ cm. Hence this mode will not propagate because $\lambda_c < \lambda_0$.

Obviously the higher TE mode will not propagate for $\lambda_c < \lambda_0$ for other values of m & n. Also for TM_{mn} mode the lowest value of m & n is unity hence no TM mode is possible at the frequency.

Since the guide wave length $\lambda_g = \dfrac{\lambda_0}{\sqrt{1-\left(\dfrac{\lambda_c}{\lambda_0}\right)^2}} = \dfrac{0.1}{\sqrt{1-\left(\dfrac{0.1}{0.144}\right)^2}} = 0.141$ metres.

We get phase velocity $v_p = \left(\dfrac{\lambda_g}{\lambda_0}\right)c = \dfrac{0.141}{0.100}\times 3\times 10^8 = 4.23\times 10^8$ m/sec.

And group velocity, $v_g = \left(\dfrac{\lambda_0}{\lambda_g}\right)c = \dfrac{0.100}{0.141}\times 3\times 10^8 = 2.28\times 10^8$ m/sec.

The phase constant, $\beta = \dfrac{2\pi}{\lambda_g} = \dfrac{2\times 3.14}{0.141} = 44.7$.

Remember

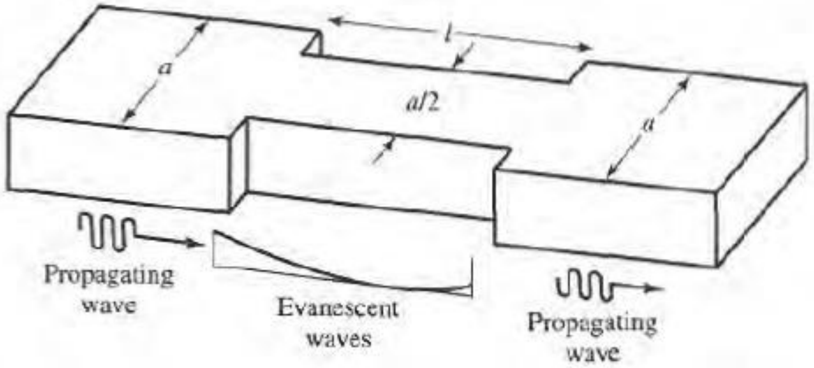
$$\beta = \frac{2\pi}{\lambda_g}$$

$$v_{ph} = \frac{\omega}{\beta} = \frac{2\pi f}{2\pi/\lambda_g} = \frac{\lambda_g}{\lambda_o} * c$$

if the medium is free space then $v_{ph} v_g = v^2$

Example 5 :

3.5 An attenuator can be made using a section of waveguide operating below cutoff, as shown below. If $a = 2.286$ cm and the operating frequency is 12 GHz, determine the required length of the below-cutoff section of waveguide to achieve an attenuation of 100 dB between the input and output guides. Ignore the effect of reflections at the step discontinuities.



(3.5) In the section of guide of width a/2, the TE_{10} mode is below cutoff (evanescent), with an attenuation constant α :

$$k = \frac{2\pi(12{,}000)}{300} = 251.3 \ m^{-1} \checkmark$$

$$\alpha = \sqrt{\left(\frac{\pi}{a/2}\right)^2 - k^2} = \sqrt{\left(\frac{2\pi}{.02286}\right)^2 - (251.3)^2} = 111.3 \ \text{neper/m} \checkmark$$

To obtain 100 dB attenuation (ignoring reflections),

$$-100 \, dB = 20 \log e^{-\alpha l}$$

$$10^{-5} = e^{-\alpha l}$$

$$l = \frac{11.5}{111.3} = 0.103 \, m = \underline{10.3 \, cm} \checkmark$$